Today we will be starting our module three on neural control. We have already finished two modules; they are neural networks and fuzzy logic. We gave some preliminary understanding about neural networks as well as fuzzy logic. We would now go deeper into control aspect. Before I go deeper into control aspect using neural network, what I would like to do is that, I will give you an overview of this field and in particular relevant citations of works that are done in neural control, important works. So, this is a complete overview of neural controller. I would not say it is an exhaustive review; it is not. But certainly, it would help you before we go in depth because, I will not be able to cover each and every aspect of neural control so, whatever I will cover today, I will select a few of them and discuss in this course.

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Particularly, I will be surely discussing the first part with what I am trying to say here; adaptive control paradigms: direct adaptive control and indirect adaptive control using neural network; these two I will surely cover. Further methods that I will be describing; it all depends which one
is more interesting for me to teach you. Nonlinear systems, we will classify them according to the way the controller has been neural controller has been designed various neural network models other neural control architectures and summary.

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Direct adaptive control - what we do normally in direct adaptive control is, given a plant and a controller, controller has certain parameters and the parameters are directly updated using the error signal here (Refer Slide Time: 03:15). That is, between the plant output and the output of a model, reference model or if it is not reference model it can be also the desired trajectory. The plant output has to follow some command signal $y_d$ or it has to follow some output reference model the output given by a reference model you give a command signal $R$ here. The error generated here is used to directly tune the controller parameters in such a way the overall system dynamics is stable. So, the parameters of controller are directly adjusted to reduce some norm of the output error and this is called adaptive control - direct adaptive control. Here we do not do any kind of identification of plant parameters directly.
Some important works on direct adaptive control using neural network; see, direct adaptive control is there in the literature for a long time but, we are only concerned about neural network based direct adaptive control. The first important paper that appeared for direct adaptive control using neural network was by Sanner and Slotine in IEEE transactions neural network in 1992, the Gaussian networks for direct adaptive control. Then the direct adaptive control paper is multilayer discrete time neural net controller with guaranteed performance; this is Jagannathan and Lewis in IEEE transactions neural network 1996. Another paper - direct adaptive neural network control for unknown nonlinear systems and its application by Noriega and Wang in IEEE transactions neural network January 1998; here they considered a non-affine system and applied to the control controller to a fluid flow system. Similarly, robust neural network control of rigid link electrically driven robots by Kwan Lewis and Dawson, appeared in IEEE transactions neural network in 1998.
Adaptive output feedback control of uncertain nonlinear systems using single hidden layer neural networks by Naira Hovakimyan, Flavio Nardim, and A Calaise, and Naikwan Kim in IEEE transactions neural networks 2002 and this the application examples were Vanderpol oscillator as well as R-50 helicopter. The novelty here is that, dynamics and dimensions of regulated system may be unknown but the relative degree of regulated output must be known. With this assumption this controller was designed neural network control of non-affine nonlinear system with zero dynamics by state and output feedback by Ge Zhang in IEEE transactions neural network 2003, they applied this to a temperature control of a thermal reactor.

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Similarly, direct adaptive control, stable adaptive neural control of nonlinear discrete time systems by Zhu and Guo in IEEE transactions neural network 2004; they applied their controller to non-affine discrete time system. They used a recurrent neural network; network weight update is derived from Lyapunov analysis connection between weight convergence and reconstruction error of the network is established and they applied the controller to liquid level liquid level system and Vanderpol oscillator. There is another paper on direct adaptive control a stable NN based observer with application to flexible joint manipulator by Abdullahi Talaivi and R V Patel in IEEE transactions neural network 2006, a very recent paper. They also designed the control for non-affine system. You must know the direct adaptive control design for affine system is easy, relatively easy and it is very difficult for non-affine system. An observer that uses
nonlinear parameters neural network; no strictly positive sphere assumption imposed Lyapunov stability, no analysis they use the Lyapunov stability analysis for validating their controller.

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Then this is a direct adaptive control. Now we will be talking about indirect adaptive control. In indirect adaptive controller, given a plant we immediately identify the plant model and we use the plant model to tune the controller parameters. The controller parameters are tuned by $E_c$ here as well as the $P$ dot the controller parameters updated is of function of $E_c$ as well as the model parameters of the system identified model parameters. The parameters of plant are first estimated and the controller is designed assuming that estimated plant parameters represent true plant parameters. Indirect adaptive control involves explicit system identification; this is very important it involves explicit system identification.
Query based model learning and stable tracking of a robot arm using radial basis function network by Behera; this is my work, computers and electrical engineering 2003.

Nonlinear systems: we will define some important classes, affine system strict feedback from singularly perturbed form non-affine system interconnected system. Normally, the way we assume the model of the system that makes a system based on that the controllers are normally
designed. Normally, as I said earlier the affine system the design of controller is little relatively easy but non-affine system; it is difficult. In interconnected system, we represent a non-affine or affine system around a nominal plant; whereas, the nonlinear terms are represented as a separate term in interconnected system.

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Let us first go to the affine system, very simple. We will be in this course actually I will focus a lot on affine system at least two classes on affine system. The normal structure of an affine system is $x \dot{} = f(x) + g(x)u$ and here the conditions are that either $f(x)$ and $g(x)$ are unknown and the other condition is that $f(x)$ is unknown and $g(x)$ is known. Normally, when $g(x)$ is known and $f(x)$ is unknown this control system design is very easy which we will see in this course, but when both are unknown it is very difficult even for affine system to design controller.

Neural networks can be used under following two classes; either $f$ is unknown, $g$ is known $f$ and $g$ are unknown. I can show you a simple way; how to design the controller using feedback linearization and how neural networks are used. Here, if I define $e$ is equal to $x_d$ minus $x$ and $e$ dot is minus $x_d$ dot minus $x$ dot and $x$ dot is $f$ minus $g$ $u$ choosing a controller $u$ equal to $1$ upon $g$ minus $f$ plus $x_d$ dot plus $Ke$. The closed error of dynamics is $e$ dot plus $Ke$ equal to $f$ minus $f$ hat $f$. If I know $f$ then this is $0$, then this is a stable dynamics. Normally, $f$ is not known and hence $f$ hat is the estimate using neural network.
Now, the objective of neural network control direct adaptive control using affine system is how I update the weights of this neural network such that my close loop system is stable. Controller is the same $u$ equal to 1 upon $g$ minus $\hat{f}$ plus $x_a$ dot plus $K_e$ structure is same. Only thing is that, how do I find the weight update law for $\hat{f}$ such that the close loop system is stable. Neural network weight update law is chosen to keep the weights bounded to clean to keep the weights bounded to make the system Lyapunov stable.

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Thus, it is a specific affine system. This is another kind of affine system which, we call strict feedback form. Consider the system in the following form; we will have one class. I will also take one example on strict feedback form because it is very interesting. So $x_1$ dot is $F_1 \ x_1$ these are all you can think. These are all each equation is a scalar differential equation and we have $m$, differential equation but each one can be also a vector differential equation; it is alright.

Let us think that they are all scalar equations. So $x_1$ dot is $F_1 \ x_1$ plus $G_1 \ x_1 \ x_2 \ x_2$ dot is $F_2 \ x_1 \ x_2 \ G_2$ $x_1 \ x_2 \ x_3$ and so on until $x_m$ dot is $F_m \ x_1 \ x_2$ until $x_m$ and $G_m \ x_1 \ x_2 \ x_m$. So, $G$s are functions where $G_1$ is function of $x_1$, $G_2$ is function of $x_1 \ x_2$ and so on until $G_m$ is function of $x_1 \ x_2$ until $x_m$ and similarly, $F$s $F_1$ $F_2$upto $F_1$ $F_1$ is function of $x_1$ $x_1$ $F_2$ is function of $x_1 \ x_2$ and so on. So, if I can represent my system in this form this is called strict feedback form so what is the advantage in this, we use back-stepping idea if it is in a strict feedback form then back-stepping principle can be used to design controller. What we do here, we assume this $x_2$ should follow a trajectory called $x_2d$ in such a way that $x_1$ will track $x_1d$. We deal with each individual subsystems individual differential equation separately. What we do here, we say $x_2$ is tracking $x_2d$ following if $x_2$ follows $x_2d$ then $x_1$ follows $x_1d$, the first one.

The second one (Refer Slide Time: 18:24) if $x_3$ follows $x_3d$ then $x_2$ tracks $x_2d$ and so on. Finally, choose $u$ such that $x_m$ tracks $x_md$ and this is the way whereby, the steps pure back-stepping
controller are designed when we know all these functions exactly $F_1, F_2, F_3, F_m$ and $G_1, G_2, G_3, G_m$ but when these $F_i$s are not known then $F_i$s are approximated using neural network. Then the neural network controller objective would be that how do I design or find the weight update rule for these neural networks because you see that there has to be $m$ neural networks to predict these functions $F_1$ to $F_m$. The weight update laws for these $m$ neural networks have to be devised in such a way the close loop aero-dynamics for this system can be stable.

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That is about affine systems now we will go to singularly perturbed model. A large class of nonlinear system can be described by the equation $x_1'$ = $f_1(x, u)$ and $\varepsilon x_2'$ = $f_2(x_1) + f_2(x_1) x_2 + g_2(x_1) u$. Here if I say $\varepsilon$ is much less than 1 then, this system of equation represent something interesting that is the dynamics of $x_2$ are much faster than that of $x_1$. There is a slow subsystem as well as fast subsystem. So this is my fast subsystem; this is my slow subsystem so we can do decomposition. Slow and fast system decomposition are done by $x_1$ is $x_1$ bar $x_1$ tilde and $x_2$ is $x_2$ bar $x_2$ tilde $u$ is $u$ bar plus $u$ tilde; the control inputs for fast and slow subsystems are designed independently of each other so for $x_2$ i design separately for $x_1$ i design separately.
Once set about affine systems, now we will go to non-affine systems where x dot is less than, it is a combined function of x and u; you cannot separate within states and control. Control and states are not at all separated and y is a nonlinear function of x and u combined or x dot is f x plus g x and u. Again here, the dynamics is complete combination of x and u. u and x are not separable; this is called non-affine system.

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Now, we come to another class of non-affine systems which we can say interconnected system where, an interconnected system of N subsystem is represented as $x_i \dot{=} A_i x_i + b_i u_i + D_i + \sum_{j=1}^{N} f_{ij}(t, x_j)$ and $y_i = h_i^T x_i$ where for ith subsystem $x_i$ is the state vector and this is a state vector $x_i$; $u_i$ is the control variable $y_i$ is the output $D_i$ is the disturbance vector and $f_{ij}$ here contains the nonlinearities of the ith subsystem and nonlinear interactions with the other subsystems. The constant matrices $A_i$, $b_i$, and $h_i$ are unknown constant matrices and nonlinear terms $f_{ij}$ are assumed to satisfy some bound that means, this as a lower bound given by $a_{ij} \| x_j \|$. 
The control objective is to design local controllers so that individual systems track the outputs of the corresponding reference model. This is my reference model because of presence of interacting functions $f_{ij}$; the overall system becomes nonlinear and difficult to control. So again, you know that we can use robust control theory for solving the problem for interconnected system. In fuzzy control theory, TS fuzzy model we will represent a system dynamics like an interconnected system; we will show that later in this course.

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Given that now we will go to neural network models we have already talked about this while teaching neural networks because we always you know describing neural network we always talked about system identification in neural network; but, this is just an overall view that we can use neural network models for function approximation feed-forward networks, multilayered networks, radial basis function networks, recurrent network, Hopfield network memory neuron network, dynamic neural network then CMAC cerebellar model articulation controller self-organizing map.
Feed-forward network - the multilayer network, one or more hidden layers; activation function is a smooth function, it has to be smooth. What you are seeing is we have already discussed that, so I will not discuss details about it. You have many layers these are all hidden layers; this is my output; this is my input. The activation function for each computational unit is this. It can be a sigmoid function for multilayered network, most widely used in neural controller output. Neurons may be linear; it is made linear for sake of advantage. Radial basis function is single hidden layer like your activation function forms a cluster in the input space and Gaussian or thin-plate. The activation function here can be a Gaussian or thin-plate spline function. There are many other functions also: quadratic, inverse quadratic and so forth and the output is always a linear; this we have already discussed, what is the radial basis function network in the neural network class.
Similarly, recurrent just like feed-forward networks have been used in control. Similarly, recurrent networks also have been used in a control full feedback connection. Learning algorithms are normally BPTT and RTRL; this you see that each neuron the outputs are fed back to each neuron. So, output why that \( y_{t-1} \) with a delay it is all fed back to the same neuron. All the outputs are connected to each neuron, then it is called fully connected neuron and the learning algorithm for such recurrent network is back proposition through time and real time recurrent line algorithm. Always the problem with recurrent network is that computations with complexity because lot of connections are there; whereas, if I take partially or locally recurrent network which you see here, this is the unit of my computer and locally it has a memory, this has a memory here, memory here, these kind of things but the connection is feed-forward. You see there is no connection from this to this and from this to this. The layer wise connection is feed-forward but, locally each computationally made is recurrent in character. So, this is called locally recurrent globally feed-forward network. This is normally also said as memory neuron network; this reduces computational complexity.
Now, dynamic neural network is also another recurrent neural network the structure can be little different where, you say that this is a single neuron. You can easily see that this is a single neuron and the output of this single neuron is a differential equation given by $x_i$ dot is $d_i x_i$. You see that $d_i x_i$ here plus this is all the weights $a_{ij}$ and phi $x_j$ is the output of these units and input is $x_1 x_n$ and so forth and finally $b_i$ into $u$ and where $d_i$ is less than 0 and phi $x_i$ this phi $x_i$ is actually it can be any kind of activation function, but normally its bipolar activation function.

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Now we will go to another category of networks that also approximates the nonlinear functions. This is called CMAC. There are lot of applications in CMAC for control which is several model articolus control. You can easily see that here this is my input vector; this called a possible long memory virtual memory and this is called working memory and you see that there is a mapping from this virtual memory to this working memory. It is a perception like associative memory with overlapping receptive field that is capable of learning multi-dimensional nonlinear functions. CMAC represent function \( y = f(x) \) using the two primary mapping; so from \( x \) to \( A \) that is the mapping \( S \) from \( x \) to \( a \) this one, when \( x \) is a continuous \( x \) dimensional input space. \( A \) is the association space and \( Y \) is one dimensional output space so normally this mapping from
A to Y is a linear mapping; so this is a schematic affair cerebral model articulus control architecture. In fact this model was proposed by Alvez way back in (37:47) seventies.

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This is a schematic affair several model articulus controller architecture this was in fact this model was proposed by (37:49) 1975 and then Miller and his group. They did a lot of work on CMAC based control some of the works that you can easily see that here a new approach to manipulator control by Alvez (38:08) in 1975 journal of dynamical systems measurement and control. Discrete time CMAC neural network control of feedback linearizable nonlinear systems under persistence of excitation by Jagannathan IEEE transaction neural network 1999. CMAC neural networks for control of nonlinear dynamical systems structure stability and passivity by Comm (38:35) Automatica April 1997.

CMAC based adaptive itself adaptive critic self-learning control by Lin Kim IEEE transaction neural network September 1991. An adaptive control system design using memory based learning system (38:51) 97. Theory and development of higher order CMAC neural network L H and G (39:00) IEEE control system magazine 1992. I am sorry that in this list the works of Miller and co who were pioneer in the work of CMAC pest control are missing in self-organizing map. Self-organizing map - this is another network by which we can develop neural model of dynamical systems. Here again, we can do any approximation of any nonlinear function.
Self-organizing is characterized by a formation of a topographic map of input pattern, in which the spatial locations of neurons in the input pattern are indicative of statistical features contained in the input pattern. A normal approach is competitive learning; a continuous input space of activation pattern is mapped onto a discrete output space of neuron Gaussian activation function.

In self-organizing map what we normally do is that, given an input space we create a cluster of the input that means if I look at the data in the input space they can be infinite through clustering what I am saying is that I am trying to represent this input data using very few number of representatives and for each representative I create a cell and within a cell I define a linear relationship between input output. A linear relationship between, from input to output is established for each discrete cell. You can think that these are all its discrete cell although my input data dimension is infinite, I have finite cells to represent this data and within a cell I establish a linear relationship like if I say my output is $y$ [Refer Slide Time: 41:42] an input is $x$ so around some $x_0$, I establish a relationship $A \cdot x - w$. $w$ is associated reference vector with each discrete cell, $x$ is my new input, $A$ is my Jacobian matrix, $y_0$ is the associated output given an input $x_0$, so $y_0$ is actually corresponding to some $x_0$. So you can easily see this is the Taylor rigid expansion. There may be other methods; use any kind of linear approximation scheme from input to output to describe a relation between from within the disk itself and then we have a
mechanism how to express the overall response using the individual responses. One of the very interesting features in self-organizing map is the neighborhood concept.

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Multiple model based flight control design Thampi Principe Motter Cho Lan IEEE symposium on circuits and systems 2002. The self-organizing map; this nice book by Kohonen book also is there but this is a paper on proceedings IEEE 1990. Facts on self-organizing map I will take two classes. I will show how we can do visual motor coordination using self-organizing map in this course.

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Neural control architectures; given all the background that I talked about, we have already discussed what is adaptive control and indirect adaptive control. Besides direct adaptive control and indirect adaptive control we can have also other control architectures. These are some of the things; although direct adaptive control and indirect adaptive control neural network has been very popular does not mean that other controls are not popular. The schemes which are model reference control, internal model.

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This particular adaptive critic based control is becoming very popular nowadays. Lot of work currently researchers are doing on adaptive critic based control because in this the concept of optimality in control design using neural network has been introduced. It is a model reference control we talked about. So here you have a plant, you have a controller and desired command signal comes from a model and a command goes to the model; plant output is compared with the model output reference output and then the controller parameters are updated. So, desired performance of closed loop system is specified through a stable reference model M the control system attempts to make the plant output $y_p$ match reference model output $y_r$ asymptotically. Controller is the inverse plant model.
Internal model based control: you say this is a plant this is my model. M represents the neural network that identifies the plant model; C is the neural network controller and F is the linear filter for robustness or for signal conditioning you can say this kind of structure is normally known as the implementation of internal model control is limited to open loop stable system. Here, the C is derived from M.

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Now, we go to the predictive control. In predictive control, a NN model, M provides the prediction of the future plant response over specified horizon the optimizer computes the control u which minimizes, following cost function. What we do is that in predictive control given the model, model predicts given a sequence of u so, what we do is that we first identify M for the plant and then we utilize C and M in conjunction in a way that we try to tell this controller to create a sequence of control in port and M would predict what should be the output in such a way that this y M should follow actual plant output - the commanded output. That way we learn; we create a predictive control assuming what should my control actions such that my model actually follows the model would respond to the desired predicted value. This is what we said the optimizer computes; the control u which minimizes following cost function. You see that this is my plant output and this is my model output and this is my controller actual and this is previous
instant control action and present instant control action. This is kind of a parameter subject to the constraint of the plant model. Another NN model C may be used to mimic the action of optimizer thereby obviating the need of a separate optimizer routine once training is complete.

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This is our final class of controller that I will be describing. Today adaptive critic based control optimality is introduced in non-controller nonlinear control. We know that optimality can be for linear systems how to design optimal controls. In adaptive critic based control prime principle is that the innovation is that optimality is introduced in nonlinear control using neural network, it incorporates reinforcement learning and dynamic programming adaptive critic utilizes an approximation of the optimal value function to accomplish its control design, approximate dynamic programming and that is why this is known as approximate dynamic programming. Dynamic programming means, that are known in the literature dynamic programming means to design optimal controller and neural network based optimal controller. In the present context they are normally known as approximate dynamic programming. An approximate dynamic programming structure consists of following main components: actor, critic, plant model and training loop for both actor and critic.
This is your adaptive critic based controller. You see the schematic of critic based controller is that you have a plant, actor means actually a controller. You have a critic 1, critic 2. So, what all these things are, what all these things are doing here, I have written here. Both actor and critic are represented by some NN models actor critic 1, critic 2; they are all neural network models.
Critic approximates the cost to go value function \( J(t) \) and \( J(t+1) \) from the state information \( x(t) \) and \( x(t+1) \). What does it mean? Given \( x(t) \) and \( x(t+1) \), given \( x(t) \) and \( x(t+1) \) and the critic has to evaluate what should be the kind of an optimal trajectory along which the plant should travel so that our cost to cost function is minimized. Actor approximates the control action based on critic feedback, the training is based on Bellman's recursive formula given by:

\[
J(t) = U(t) + \gamma J(t+1)
\]

This is my cost function that is updated \( J(t) \) is \( U(t) + \gamma J(t+1) \) actors weights are updated, so as to minimize the present value function \( J(t) \) critic weights are updated, so that the cost function represents the actual cost to go function \( J(t) \).

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This is adaptive critic based control design which you saw here. This particular scheme at the moment is gaining momentum because it is interesting to see if we can design optimal controllers for nonlinear systems as well because you see that the usual Bellman Jacob formula to apply for nonlinear system and derive control law is very difficult. That is why researchers are trying to find out simpler methods by which they can introduce optimality in control.

Implementation of adaptive critic based neural controller for turbo generator in a multi machine power system by Vinayagamoorthy Harley and Wunuch IEEE transaction neural network 2003 Jagannathan and Salan IEEE transaction neural network March 2004. So that gives you an overall idea about where we stand as far as neural control is concerned.
To tell you final concluding comments, what we discussed is that we discussed in intelligent control framework in neural control framework; what are direct and adaptive control schemes that is, direct and indirect adaptive control schemes. We gave the references, we classified the nonlinear systems and for each category, we showed what are the relevant works that have been done in the net literature. We also provided some neural network models that have been used; popular neural network models that have been used in control literature. We also gave an overview of some of the control neural besides direct and indirect adaptive control schemes. The other neural control architectures and in which specifically adaptive dynamic programming is becoming very popular and with all these schemes where details were given were presented with detailed literature survey. I hope this lecture should help you to go deeper into the subject on neural network. Thank you.