Fuzzy Lyapunov controller computing with words, this is the lecture 4 of this module 4 on fuzzy control. As I said in the last class that given the mamdani type of fuzzy logic controller, the membership functions are optimized using genetic algorithms.

But the most important aspect of mamdani type of controller is to generate a proper rule base. Here, of course you can also generate rule base using genetic algorithm but today, we will be learning how to generate a rule base using the concept of Lyapunov function. So this is the topic today, fuzzy Lyapunov controller computing with words.

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Topics to be covered today are classical Lyapunov synthesis; Fuzzy Lyapunov synthesis; rotational proof mass actuator system, this is the physical system on which you will be testing our algorithm; construction of rule base for RTAC, the short form is RTAC here,
RTAC using fuzzy Lyapunov synthesis: regulation problem; controller design for regulation; construction of rule base for RTAC using fuzzy Lyapunov synthesis: tracking problem; controller design for tracking; and finally, simulation results.

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Fuzzy Lyapunov synthesis: an introduction- the classical Lyapunov synthesis method is extended to the domain of computing with words. This new approach is used to design fuzzy controller. Assuming minimal knowledge about the plant to be controlled, the proposed method enables us to systematically derive the fuzzy rules that constitute the rule base of the controller. The objective is in this method, how do we obtain fuzzy rule base given the system, some prior in knowledge of the system or approximate model of the system, how do we generate a rule base?

This approach is demonstrated by designing Mamdani-type and Takagi-Sugeno-type fuzzy controllers for nonlinear plants.
The most difficult aspect in the design of fuzzy controllers is the construction of the rule base. Fuzzy Lyapunov synthesis is based on extending the classical Lyapunov synthesis method to design of fuzzy controllers. It enables to systematically derive the fuzzy rule base. Based on classical Lyapunov synthesis method, we construct a Lyapunov function candidate $V$ and then determine the conditions required to make it a Lyapunov function of the closed loop system. Since we assume fuzzy knowledge about the plant to be controlled, the derived conditions can be stated as fuzzy if then rules.
So now let us go to the system description. Consider the following single-input, single-output system. \( x \) dot equal to \( f(x, u) \), \( y = h(x, u) \) where \( f \) and \( h \) are nonlinear functions. This is \( n \) dimensional state space, so \( f \) is a \( n \) dimensional vector; and \( u \) and \( y \) they are the input and outputs of the system respectively; \( x \) \( n \) dimensional system \( x_1, x_2 \) until \( x_n \), \( \mathbb{R}^n \) is the state vector of the system. The control objective is to stabilize the system around some operating point \( x_0 \). Without loss of generality we may assume \( x_0 \) equal to 0.
So Lyapunov stability theory which we have been talking again and again, I am just revisiting that, the system $x \dot{} = f(x, u)$ is Lyapunov stable around the operating point $x = 0$. If there exists a continuously differentiable function $V(x)$ which is known as Lyapunov function such that the following requirements are met. $V(x)$ is positive definite in the neighborhood of 0 and $v \dot{}(x)$ is negative definite in the neighborhood of 0. If we can ensure that then the system is Lyapunov stable.

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So fuzzy Lyapunov synthesis - assume that the exact system model is unknown but we do have some partial fuzzy knowledge about the system. As in classical case we consider a Lyapunov function candidate $V$, derive an expression for its derivative and then obtain the fuzzy rule base for the control input $u$, so that $v \dot{}$ is negative definite. Based on the rule base, a fuzzy controller $u$ is obtained using general inference mechanism and defuzzification method.

What is the objective here is that we prescribe a Lyapunov function for the system and then we construct, find the right derivative of the Lyapunov function, substitute the dynamics of the system in that and now we create a rule base in such a way this Lyapunov function is qualitatively negative definite, not quantitatively. So what is fuzzy
Lyapunov synthesis is we are using a qualitative negative definite of $v$ dot. This is the difference qualitative negative definiteness of $v$ dot, instead of a quantitative negative.

The actual Lyapunov function approach to test the stability of the system is to find quantitavely $v$ dot is negative definite. But now we are talking about a concept qualitatively, if we can make $v$ dot to be negative definite.

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FLC structures of the rule base, so normal we have two kinds of we can formulate any fuzzy logic controller of mamdani type. We can have two categories of fuzzy rule base. The representation 1 is if $x_1$ is $A_1$ and or $x_2$ is $A_2$ and or $x_n$ is $A_n$ then $u$ is $B$ where $A_i$’s and $B$ are linguistic variables large or small. So $x_1, x_2$ and $x_n$ they are either state variables or error in state variables.

Representation 2 is if $x_1$ is $A_1$ and or $x_2$ is $A_2$ and so on or $x_n$ is $A_n$ then $u$ is a definite function here the $B$, $u$ is $B$ are the fuzzy linguistic variable where in the second representation $u$ is an exact function. $u$ is $f(x_1, x_2, \ldots, x_n)$ where $f$ is a linear function; this can be also nonlinear function, $f$ can be also linear function here, normally is a linear function.
So fuzzy Lyapunov synthesis: motivation- what is the motivation here? In a general fuzzy logic controller, the control problem is to design a fuzzy controller using information based on some physical intuition even if the exact system dynamics is not known.

But the main problem is constructing the rule base for the controller. Rule base is obtained using the notion of classical PD or PID controller in general for mamdani type of fuzzy logic controller. But in a fuzzy Lyapunov synthesis a rule base is obtained using the notion of Lyapunov stability. So this is the prime motivation. I can tell you how normally we do a fuzzy rule base, if this is my general characteristic of a second order system. So what I would like, when I describe this response, this is my t, this is my y

I would like when I am here I should come very fast to this line, which is my desired command signal 1.0. What I will say, when my error is small and the derivative of error, the rate of change in error is either small or large whatever it is, when it is small I would like it to come fast. My controller must be very large. I can write for this kind of PD or PID type of controller the rule base is if e is small in this zone and e dot is small or large then u is large. By simple commonsense I can write.
Similarly, if I say e is small here and if e is small in this zone and e dot is also small then u is small because I have already reached almost I am reaching the set point. But if e is small and e dot is large, so obviously I must reduce the control action, so that is, in that case I have to say u is negative small, instead of positive small. Like that the normal rule base is done in case of PD or PID type of controller.

But today we will see a very systematic method of generating rule base. Actually this entire class today is to let you know or we will learn how to generate this rule base systematically using Lyapunov stability theory. Before that, I will just explain to you one physical system on which will be applying this thing, this is called rotational translational proof mass actuator.

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You can actually see this is my rigid surface to which a cart is bound using a spring. Inside the cart there is the pendulum which rotates by the action of the torque and the pendulum link length is l and there is an external disturbance f that is applied on the cart. See the objective is to control this pendulum the way we like.

I may like to keep this pendulum at an angle theta or vertically downward vertically upward, whatever is the desired thing or it has to follow certain trajectory, so whatever
may be the requirement this is a typical control system because it is nonlinear and also it a bench mark problem.

The RTAC system, consider the following example rotation translational proof mass actuator RTAC. The RTAC system combines a translational oscillator with a rotational proof mass actuator. This particular portion now oscillates by the action of the spring. So this called combines a transitional oscillator. This oscillates in the x direction with a rotational proof mass actuator. This is rotation. The oscillator consists of a cart connected to fixed wall by a linear spring. The cart is constrained to move in the x-direction only. The proof mass actuator is attached to the cart and is controlled by an applied torque N. F is disturbance force that perturbs the cart. F is a disturbance force that gives such a disturbance input.

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The cart position and actual angle is $x_c$ and $\theta$ respectively, that is the cart which you saw here this is $x_c$ and this theta already which have already shown. Mass of the cart is given 1.36 Kg; proof mass is 0.096 that is the mass of this cart is $M$ and proof mass is small m. So small m is 0.096 Kg; distance of the link length of the pendulum is 0.059 meter; Moment of inertia of the proof mass is I is 0.00022 Kg meter square; Spring constant k is given 186.3 Newton per meter.
Now we define some normalized variables which is \( \zeta = \sqrt{\frac{M + m}{I + mL^2}} \); \( \tau = t \frac{K}{M + m} \); and \( u = \) this is my actual input and \( \frac{M + m}{K} \) into \( 1 - \frac{mL^2}{I + mL^2} \); \( K \) is the spring constant; and \( \omega = \frac{1}{M + m} \) by \( I + mL^2 \).

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Given that, you see that we can write, we can derive the dynamics of the RTAC system, you can easily see that it has four states. \( x_1 \) is \( \zeta \) actually represented \( x_c \) normalized; this is \( x_c \) dot; the velocity of the cart normalized which is \( x_2 \); and \( \theta \) is the pendulum angle \( x_3 \); \( \theta \) dot is the angular velocity of the pendulum which is \( x_4 \); and control variable is \( u \) which is also normalized; \( n \) the torque; and disturbance is \( w \), so this one \( w \) is the disturbance.

If we look at \( x \) dot this is the fourth order system and the expressions are \( x \) dot equal to \( x_2 \), the advantage is that we have not kept any parameters associated, this is simplified \( x_2 \), you can see \( x \) dot is in this, this I can say, this is simply a function of \( x \), this is another function of \( x \) may be \( g(x) \), and this is \( d(x) \).

You can say this is a function vector with four dimensions, with function vector with another four dimensions and another function vector with four dimensions. You can see that system is highly nonlinear if silent is defined as \( mL \) upon root over \( I + mL^2 \)
into M plus m, you can easily derive this equation, I will not discuss further on this. All that I am trying to tell you is that RTAC system has a nonlinear dynamics which has type $x$ dot is $f(x) + g(x) u + d(x) w$, where $w$ is the external disturbance; $u$ is actual control input.

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The fuzzy Lyapunov synthesis of RTAC- the system dynamics now can be written as, as we said $x$ dot is $f(x) + g(x) u + d(x) w$. Without knowing $f(x)$, $g(x)$ and $d(x)$ following linguistic statements can be formulated. Even if I do not know what is $f(x)$ $g(x)$ $d(x)$ exactly, I can still write the state of the system is described by $x_1$ the cart’s position; $x_2$ equal to $x_1$ dot; and $x_3$ is actuator’s angle; and of course $x_4$ is $x_3$ dot. That is $x_3$ is theta, $x_4$ is theta dot, $x_1$ is $x_c$ normalized, $x_1$ is $x_c$ normalized and $x_2$ is $x_1$ dot. This is another important thing that I can easily tell you, $x_4$ dot is proportional to the control input $u$ that you can easily see here.
Because you see that this is the my pendulum and if I assume that there is no friction and other things here, then my pendulum, the acceleration of this pendulum, angular acceleration obviously is directly proportional to the torque applied N and since my input u is normalized value of N and hence I can always say that x₄ dot, the acceleration of the pendulum is proportional to the control input u.

Similarly, x₂ dot the velocity of the cart is proportional to the negative of x₁, you can easily see that because the spring is connected so it is like this, spring and this is the block mass. You can see that x₂ dot the velocity is always, because of spring action, it will be always in this direction the velocity. If I move x in this direction, x₂ the position in this direction, the velocity is in the negative direction. So x₂ dot in this is proportional to minus x₁.
The control objective is to find \( u \) such that the system is stable around \( x = 0 \). This is the regulation problem. Let \( V = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2) \) be the Lyapunov function candidate. Then using S-1, the assumption that we have four states \( V \) dot is \( x_1, x_1 \) dot and \( x_1 \) dot is \( x_2 \). Similarly, this is \( 2x_2 \) \( x_2 \) dot, so \( x_2 \) \( x_2 \) dot plus \( x_3 \) \( x_3 \) dot and \( x_3 \) dot is \( x_4 \) and similarly, this is \( x_4 \) into \( x_4 \) dot. Before this is the second step, the first step \( V \) dot if you look at here is \( x_1 \) \( x_1 \) dot plus \( x_2 \) \( x_2 \) dot plus \( x_3 \) \( x_3 \) dot plus \( x_4 \) \( x_4 \) dot, because we know already that \( x_2 \) is \( x_1 \) dot and \( x_4 \) is \( x_3 \) dot, so this second step comes.

Now you see that, we have already said \( x_2 \) dot is proportional to minus \( x_1 \). Obviously, qualitatively if I put \( x_2 \) dot is minus \( x_1 \), these two terms qualitatively cancels off, where as only these two terms remain, \( x_3 \) \( x_4 \) is remained and I know already that \( x_4 \) dot is proportional to \( u \), so instead of \( x_4 \) dot I put \( u \).

You see that, what is the advantage here is that I am designing a rule base but while designing rule base I am not utilizing the system dynamics. I am finding how to what \( V \) dot is equal to \( x_3 \) \( x_4 \) plus \( x_4 u \). While finding this equation, I am not using the system dynamics. This is very important, system dynamics is not used.
What is used is the simple physics of the system is that, here we use $x_2$ equal to $x_1$ dot obviously the velocity $x_2$ and $x_1$ is positioned and similarly, here we use the notion that the $x_4$ is $x_3$ dot, so $x_4$ is theta dot and $x_3$ is theta, which is very simple and also we used other rule from the systemic connections.

We know already that the acceleration of the pendulum is proportional to the input and $x_2$ dot is proportional to minus $x_1$, just by constructional feature. Using these two rules, we found $V$ dot to be qualitatively $x_3$ $x_4$ plus $x_4$ into $u$. Now, even if you do not know the system dynamics, looking at this figure we can create a rule base for which this $V$ dot can be qualitatively negative definite.

So classical Lyapunov synthesis have design of view that will guarantee $V$ dot is negative definite. Same idea is applicable here but with the domain of computing with words means, this is qualitatively not quantitative. What we are trying to do is we are applying Lyapunov synthesis to the system dynamics partially known system dynamics for generation of rule base.

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Now we take the example, when I say so $V$ derived $V$ dot equal to approximately qualititatively $x_3$ $x_4$ plus $x_4$ into $u$. This can be made negative definite if the rule base is formed as follows. You see that I will take care that you understand is because this is
what you should learn how do generate a rule base, you see $x_3$ is negative. If $x_3$ is negative and $x_4$ is also negative, that means this quantity is positive. Now $x_4$ is negative and if $u$ would be come negative then this whole thing is positive, so no way you can make the system is stable. Obviously, the rule base should be $u$ to be positive, that is the rule.

I repeat if $x_3$ is negative and $x_4$ is negative, in that case $x_3 x_4$ is positive, this implies $x_4$ into $u$ should be negative to make these two quantities to be qualitatively negative. So $x_4 u$ should be negative, $x_4$ is already known as negative, so $u$ has to be positive. This is first rule. If $x_3$ is negative and $x_4$ is negative then $u$ is positive. Similarly, if $x_3$ is positive and $x_4$ is positive. $x_3$ is positive and $x_4$ is positive means again $x_3$ and $x_4$ is positive in this case. And this implies again $x_4$ and $u$ should be negative and since $x_4$ is positive so $u$ has to be negative which is negative. The second rule is if $x_3$ is positive and $x_4$ is positive then $u$ is negative. I hope you are able to follow what I am presenting you.

Now let us see the other possible combination. If $x_3$ is negative and $x_4$ is positive, in this case $x_3 x_4$ is negative. If $x_3 x_4$ is negative I must ensure that $x_4 u$, since it already negative this should be 0 is sufficient, if I make $x_4$ is 0 because $x_3 x_4$ is negative is very good. This is simple because there are other ways also is $x_3 x_4$ is negative I can say $x_4 u$ is also negative much better. But this is simplest one $x_4$.

$x_3$ is negative and $x_4$ is positive then $u$ is 0, you can also think another possibility, because $x_4$ is positive $u$ is negative, this is also possible. Similarly if $x_3$ is positive and $x_4$ is negative then again $x_3 x_4$ in this case $x_3$ and $x_4$ sorry $x_3$ negative, $x_3$ is positive and $x_4$ is negative, $x_3 x_4$ is negative. So this implies you can make $x_4 u$ is 0 or $x_4 u$ again negative. Since $x_4$ is negative so $u$ is 0 or $u$ is positive, but in this case we have only considered this 0.

I wish this is the most important part of this lecture, first of all using partial knowledge of the system and without taking the help of system dynamics we first of all found out the rate derivative of the Lyapunov function is approximately qualitatively equal to $x_3 x_4$ plus $x_4 u$ and using this expression we computed set of rule base such that $V$ dot can be negative definite.
What we have not until now found out in this is the parameters negative, what are the parameters associated with negative. I am assuming this fuzzy linguistic variable to be Gaussian functions, which we have taken in this case. Gaussian function is normally represented by two parameters— one is mean and another is sigma. If I say sigma is 1 then these linguistic variables they are all represented by one parameter that is the mean and the u is positive or negative that we have defined here, we have taken them to be single term.

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Standard Gaussian membership functions are used to define the linguistic terms in the rule base as μ positive of x is e to the power minus x minus x whole square, this is the parameter that we have to identify. Similarly, μ negative membership function of negative of the variable x is e to the power minus x plus x whole squared. And μ z x is 0 e to the power minus x square.

So x represents mean, x stands for state variable, p, n, z denotes positive, negative and zero respectively. Using the product for AND inference and “center of gravity” method for defuzzification the fuzzy controller is given as we have four rules. So for four rules the u if you look at here what I will do is that in this I find out the membership function of this, the membership function of this. So this is μ x₃ into μ x₄ so that is the membership
function for my control action and this is the fuzzy singleton which I define as $a_u$, the fuzzy singleton, therefore my control action is $a_u$.

$\mu x_3, \mu x_4$ into $a_u$ is the output of the first rule. Here, again to let to know I will again explain, here how we are actually doing this I do not have to do every thing again you can easily understand, what is control action from this rule base? Obviously, I find out what is $\mu$ and $x_3$ and $\mu$ and $x_4$, the membership function of this into $u$ is positive and this is a singleton and that is why this is $a_u$.

$a_u$ is the singleton value associated with this control action $a_u$, simply multiplied. This is the output of this controller, similarly you can easily see this is output of the rule base, this rule and the second rule output will be $\mu p x_3$ into $\mu p x_4$ into this is negative, so minus $a_u$. That is what we doing here.

So $\mu$ and $x_3$, $\mu$ and $x_4$ $a_u$, $\mu p x_3$, $\mu p x_4$ minus $a_u$ I will not have been considered this because we are assuming. Thus you see that this is 0 if I simply say the control action is 0, because control action is 0 this is sufficient for me to make this rate derivative to be negative definite. Simply making that what advantages I am gaining is that, I am not reducing the parameters that has to be optimized or heuristically that has to be tuned. This is my control action, by using center of gravity method you just add all these individual membership functions associated with control action.
So there are 4 and finally, you get the value by taking \( \mu_p(x) = e^{-(x-a_1)^2} \) and \( \mu_0(x) = e^{-(x-a_2)^2} \), we can write

\[
\begin{align*}
\mu(x) &= \mu_p(x) \mu_0(x) \\
&= e^{-(x-a_1)^2} e^{-(x-a_2)^2} \\
&= e^{-(x-a_1)^2-(x-a_2)^2} \\
&= e^{-(x-a_{12})^2} \\
&= e^{-(x-(a_1+a_2)/2)^2} \\
&= e^{-(x-x_0)^2} \\
&= e^{-(x-x)^2},
\end{align*}
\]

This is your control law. This is nothing to do because we have taken Gaussian function. Finally you can simplify this \( u \), but you also can take any other function triangular function or whatever function you want to like, but if you look at this control action what are the parameters that you do not know, this one is \( a_u \), another is \( a_{x3} \), another is \( a_{x4} \), all other things \( x_3 \) \( x_4 \), these quantities that you are getting feedback from the system. So these three parameters they have to be found and we can do any kind of optimization algorithm.
And particularly we have used genetic algorithm, particularly univarite marginal distribution algorithm. We have already described in the previous classes what univarite marginal distribution algorithm is.

This has been used to minimize this cost function and by minimizing that, the optimal parameters are found to be the mean of the linguistic variable for $x_3$ is 0.73 and for $x_4$ is 0.25, this either positive or negative. So this is the mean and $a_u$ is control action singleton is 1.4. This was done, so this is design is over, now we can simulate the system which will show the results at the end of the section.
But here, before that I will derive again another kind of rule base for the same system. The \( u \) instead of earlier case was simply a linguistic variable, but now I am assuming this is a state variable technique, where \( u \) is minus \( k_1x_3 \) minus \( k_2x_4 \). Of course this is just to simplify but actually I should have taken all the \( x_1x_2 \) I am not taking, just to simplify because if I am able to control using only two variables, I can only expect that using \( x_1 \) and \( x_2 \) I can improve the performance. So putting expression of \( u \) in \( V \) dot is already we have found out this earlier because we considering same RTAC system.

So for RTAC system giving the physical principles \( V \) dot qualitatively comes out to be \( x_3x_4 \) plus \( x_4u \). Now this \( V \) dot can be written as \( x_3x_4 \) and then \( x_4u \) is this quantity which is minus \( k_1x_3 \) minus \( k_2x_4 \), and this can be again simplified, this is 1 minus \( k_1 \) into \( x_3x_4 \) minus \( k_2x_4 \) whole square. So how do I now derive the rule base for this kind of structure?

I just have to make sure that I have to select \( k_1 \) and \( k_2 \) in such a way that this quantity is negative definite. This is what, if \( x_3 \) is negative and \( x_4 \) is negative then \( u \) is, you see that \( x_3 \) is negative and \( x_4 \) is negative, so this quantity is positive. Since \( x_4 \) is negative, the \( x_4 \) square is positive. All that I have to make sure that 1 minus \( k \) has to be negative as well as \( k_2 \) is also a positive. So \( k_2 \) is positive and 1 minus \( k_1 \), because this is positive, this has to be negative, 1 minus \( k_1 \) is negative means \( k_1 \) is greater than 1. This is my first rule.
The second rule is if $x_3$ is positive and $x_4$ is positive, this quantity is positive, this quantity is positive, to make it again negative definite, again I will have the same principle whatever I got here I must be that, $k_{2pp}$ greater than 0, $k_{1pp}$ is greater than equal to 1. Same condition, only this $nn$ stands for the rule base when $x_3$ is negative and $x_4$ is negative and here $pp$ stands for $x_3$ is positive and $x_4$ is positive.

In the second rule you saw that, the same condition came but these are different parameters you have to remember $k_{1pp}$. If $x_3$ is positive and $x_4$ is positive then $u$ is equal to minus $k_{1pp} x_3$ minus $k_{2pp} x_4$ where I showed you that this has to be positive and $k_2$ has to be positive.

Now let us go to the third one, $x_3$ is negative AND $x_4$ is positive THEN $u$ is minus $k_{1np}$ $x_3$ minus $k_{2np}$ $x_4$. How do I find out what is $k_1$ and $k_2$? It is simple; $k_{1np}$ you see that, $x_3$ is negative and $x_4$ is positive here, that makes this is negative. This is negative means this has to be positive, 1 minus $k_1$ has to be positive, so $k_1$ should be less than 1. That is $k_{1np}$ is less than 1 and here this has to be again positive quantity $k_2 x_4$ because this is negative is already there, this $x_4$ is positive so this is always positive whether $x_4$ is negative or positive. To make it positive $k_2$ also has to positive, so this is what $k_{2np}$ this has to be positive.

Similarly, if $x_3$ is positive and $x_4$ is negative, again this is negative, this has to be again positive, and again $k_1$ should be less than 1. So $k_{1pn}$ is less than 1 and $k_{2pn}$ has to be again here this is positive, so $k_2$ has to be always positive. You can easily see that this $k_2$ always in every case is positive, only in this case, the first one it is positive and the second that is negative. What you got an idea that, what should be this value $k_1$ and $k_2$, at least the magnitude?

Now *heuristically* you have to tune these values what should be $k_{1nn}$ $k_{2nn}$ $k_{1pp}$ $k_{2pp}$ $k_{1np}$ $k_{2np}$ and $k_{1pn}$ and $k_{2pn}$ heuristically, or I can use the genetic algorithm or we have done that univariate marginal distribution algorithm to optimize this parameter.

You see that considering the same membership function and using product for AND inference and center of gravity method for defuzzification, the fuzzy controller is given is
like this. Since $\mu_p$ is this and $\mu_n$ is this, which we have already done in the previous case, we can write $u$ as like this, we have a controller and by simplifying that we have actually ten parameters to identify.

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Two parameters $x_3x_4$, the mean for linguistic variable $x_3$ positive, for $x_3$ positive and $x_3$ negative the mean is $x_3$ and actually in case of one case mean is positive and in one case mean is negative. That is what we are doing that when it is negative, mean is negative, so the sign changes that is why you saw that here.
In this case this is positive $x$ when it is negative mean is negative, so it is positive quantity and mean is positive here in case of positive, so it is negative here. So $ax_3$ is 0.25 here, doing optimization using univariate marginal distribution $ax_3$ is 0.25 and $ax_4$ is 0.12 and the parameters, the controller parameters that is $k_1$ and $k_2$ for all the different fuzzy rules, four rules are there and for each rule we have two parameters $k_1$ and $k_2$ and we have got these values 1.1, 1.4, 0.6, 0.8, 0.35, 0.2, 0.4, 0.25. These parameters we derived using univariate marginal distribution algorithm.
You see that simulation results $x_1$ converges to 0 with time, what we started with we gave some initial disturbance and using the initial disturbance we are showing now, here $x_1$, $x_1$ is the position of that cart, you give some little disturbance to the cart finally it should come to the rest position.

What is the meaning of PD? This is the FLC controller where we have used the PD type of rule base that is if $x_3$ is positive and $x_4$ is positive then $u$ is negative, in that kind of rule base. This is PD; it is actually the rule base that we formulated. That is, we are denoting we are saying this is PD and this we are saying state feedback because you see that $u$ is a function of states. By doing that you see that, we have got the response that, this is in second unit, after 40 seconds almost the oscillation comes to an end.
And here $x_3$ is plotted which is the angular position, that also should come to, because the cart was disturbed, although the pendulum was in the beginning initial position, but it will start oscillating and that oscillation will gradually slow down and it equals in same amount of time this is reduced to 0, both the PD type of rule base and state feedback type of rule base and this is the control action and this is control action you see that the control action is also fairly smooth both for PD and as well as state feedback.

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Saying that what we did is the regulation that is given the RTAC system we gave some disturbance and we showed that this controller is able to stabilize the system. Now the tracking controller that we would like that our pendulum should track it as a trajectory. For tracking a desired trajectory, what we would like to have is that instead of the Lyapunov function which we had half $x_1$ square plus $x_2$ square plus $x_3$ square plus $x_4$ square for regulation problem, now we will consider $e_1$ square plus $e_2$ square plus $e_3$ square plus $e_4$ square be the Lyapunov function where $e_i$ is desired state trajectory minus the actual state trajectory. So $x_i$ is the higher state and $x_i,d$ is the corresponding desired state trajectory.

The control objective is to find a control action such that the actuator angle will follow any given desired trajectory. Hence the Lyapunov function has been changed from $x_1$ square plus $x_2$ square plus $x_3$ square plus $x_4$ square to in this case $e_1$ square plus $e_2$ square plus $e_3$ square plus $e_4$ square. I hope this is clear to you.

Now in the same manner we can also show $e_i$ dot is $x_i$ dot d minus $x_i$ dot is same as $x_i$ plus 1 d minus $x_i$ plus 1 which is $e_i$ plus 1. What is $x_i$ plus 1? Because we have the parameters $x_1, x_2, x_3, x_4$, so when I say $x_i$ plus d, when I say $x_i$ is this is $x_1$, $x_i$ dot d is same as $x_2$ d. Obviously, because $x_2$ and $x_1$ the relation is $x_2$ is $x_1$ dot. By that principle you are able to see that $x_i$ dot d is $x_i$ plus 1 d which I am writing here and similarly also I can write $x_3$ dot d is $x_4$ d, $x_3$ because $x_3$ dot is $x_4$. So obviously $x_3$ dot d is desired is $x_4$ desired.
For \( i \) equal to 1 and 3, that is what I have done, \( i \) equal to 1 and 3 and differentiating \( V \) we had \( V \) dot is \( e_1 \) \( e_2 \) plus actually this is \( e_1 \) \( e_1 \) dot \( e_2 \) \( e_2 \) dot \( e_3 \) \( e_3 \) dot \( e_4 \) \( e_4 \) dot. We already know what is \( e_2 \) is \( e_1 \) dot and this is retained and \( e_4 \) is \( e_3 \) dot. This is our actual expression of \( e_1 \) dot.

Just like this again \( x_2 \) dot is approximately minus \( x_1 \) and \( x_4 \) dot is approximately \( u \), thus in the same similar logic we can simplify this \( V \) dot to be qualitatively \( e_3 \) \( e_4 \) plus because qualitatively this will cancel out this because \( e_2 \) dot would be minus of \( e_1 \). This will be cancelling out qualitatively, what is remained is \( e_3 \) \( e_4 \) plus \( e_4 \) and \( e_4 \) dot we see that \( x_4 \) dot is directly proportional to \( u \). We can write that this is \( e_4 \) and \( e_4 \) dot is \( V \) where \( V \) is \( x_4 \) dot \( d \) minus \( u \).

This is very simple because \( e_4 \) dot is \( x_4 \) dot desired minus \( x_4 \) dot and since this is \( x_4 \) dot I put this \( x_4 \) dot to be \( u \), I can replace that by \( u \) and \( x_4 \) d minus \( u \) I can write \( V \) and that \( V \) is \( x_4 \) dot \( d \) minus \( u \).

What we have to do? I can create a rule base from this Lyapunov function assuming \( V \) to be my control input and from \( V \) if I know \( V \) then I know \( u \), actual control action, this is pseudo control action, my actual control action would be \( x_4 \) d minus \( u \).
In general what we learnt is that for regulation the qualitative equation for right derivative of Lyapunov function $x_3x_4$ plus $x_4u$ for tracking you found out this to be $x_4V$ dot is qualitatively equal to $e_3e_4$ plus $e_4$ into $V$ where $V$ is $x_4d$ minus $x_4d$ dot minus $u$ and $u$ is the actual control action and $V$ is the actual control action.

This $V$ can be designed the same way as that $u$ in the regulation problem; I will not be going in detail because this and this has the same structure. The rule base either using the PD type of controller or the state feedback controller the two rule base will have same form, only thing that I am computing $V$. After I compute $V$, $V$ will have the same structure that we earlier derived the same way but given $V$, I find out $u$ to be $x_4$ dot desired minus $V$.

Doing that and also the parameters in the first case will have three parameters and in the second case similarly ten parameters that have to be optimized.
By optimizing these parameters, we get $a_v$ not $au$, $a_v$ is 1.5, $a_{e3}$ 1.23, $a_{e4}$ is 0.55 and for this tracking controller two you have these parameters .37, .26 and these are the parameters for the state feedback controller for four rules.

So $k_{1nn}$ is 1.3 and $k_{1np}$ is .9, $k_{2nn}$ is .6, $k_{2np}$ is .45 and so on.

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Given that, the tracking result is $x_3$ is plotted against time desired PD and state feedback give the same result here for the tracking. Tracking is not plotted because you see that the rule base is very small that we have taken and also we have in this state feedback control are also we have rejected in the state feedback the input of $x_1$ and $x_3$ the state feedback for $x_1$ and $x_2$.

The tracking here is not right, some loss here otherwise tracking is properly and the control action you can see both PD and state feedback they behave almost similarly.

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Finally the summary, in this lecture following topics have been covered a classical Lyapunov synthesis method is extended to the domain of computing with words that means qualitatively how can we use a rate derivative of the Lyapunov function in such a way that the rule base can be generated. This new approach is used to design two different types of controllers actually, the two different types of rule base, controller is same fuzzy logic controller mamdani type.

Rotational proof mass actuator system is presented for simulation. Assuming minimal knowledge about the system we have systematically derived fuzzy rules that constitute the rule base of the controllers. Using rule base controllers have been designed for both regulation and tracking purposes. Simulation results are presented.
References in this case are some of the related papers are you can easily see Fuzzy Lyapunov based approach to design the fuzzy controllers, Fuzzy sets and system in 1999 by Margaliot and Langholz and same author also have a paper on fuzzy control of a Benchmark problem; computing with words approach, IEEE transaction Fuzzy Systems, 2004.

And also we have a paper some work on the directional intelligent controls schemes for a redundant manipulator presented in a conference held in Pune in 2005.

Thank you very much.